Biophysical Chemistry 3 (1975) 241–248
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SYNCHRONIZATION IN TWO INTERACTING OSCILLATORY SYSTEMS

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Received 30 January 1975

Nonlinear phenomena arising from the interaction of two oscillating systems of chemical reactions are studied experimentally. The system of two connected flow-through continuous stirred tank reactors (cells) with controlled exchange of reaction mixture is used. The Belousov reaction (oxidation of malonic acid by bromate in sulphuric acid with ceric/cerous ions as catalyst) served as model system. The frequency of oscillations was controlled by change of the reaction temperature. Phenomena such as synchronization of oscillations at a common frequency, synchronization at multiples of a common frequency, rhythm splitting and amplitude amplification were observed, depending on the degree of interaction and the differences in the original oscillation frequencies. Mathematical modelling of the above phenomena failed, probably due to insufficient knowledge of a kinetic model.

1. Introduction

Populations of coupled biochemical oscillators are a model which has often been used for theoretical explanation of rhythms, spatiotemporal control of various developmental processes, contact inhibition of cell divisions and other phenomena in living organism, see, e.g., the review by Nicolis and Portnow [1]. Basic assumptions involved in the construction of the models are:

(a) individual oscillators are nonlinear and of the limit cycle type;

(b) interaction between oscillators is weak [2,3]. The smallest system where characteristic phenomena can be studied, is a system of two oscillators. Thus Landahl and Licko [4] simulated the course of oscillations in the system of two oscillators on an analog computer, with passive exchange of matter, where oscillating biochemical reactions including three characteristic components took place. They observed that, with the increase of the degree of interaction, the oscillations in the system with slower frequencies were driven by the oscillations in the system with higher frequencies. Ruelle [5] has shown, that due to interaction of two oscillating systems (periods of oscillations T_1 and T_2) oscillations with the periods mT_1 and nT_2 can arise, where m and n are small numbers and $n/m \sim T_1/T_2$.

Burton and Canham [6] have considered the behaviour of coupled oscillators from the point of view of contact inhibition of cell divisions. They have shown, that the time course of the concentrations of characteristic components is determined by the value of the parameter $k_i = k/T_i$, where k describes the intensity of exchange between the interacting cells and T_i is a characteristic period of oscillations in the ith cell. The authors have also shown that in the system periodic amplification of amplitudes of oscillating components can occur. Several papers by Russian authors were devoted to the problems of determining the conditions for synchronization and rhythm splitting [7–9]. They have illustrated with different examples that possibilities for synchronization increase with increasing degree of interaction and decreasing difference between frequencies of interacting oscillators, in agreement with ideas proposed earlier by Winfree [3].

All the above discussed treatments were purely theoretical. In our work we have aimed at experimental verification of existence and nature of the above mentioned nonlinear phenomena. Two continuous, well stirred, flow-through cells (reactors) connected

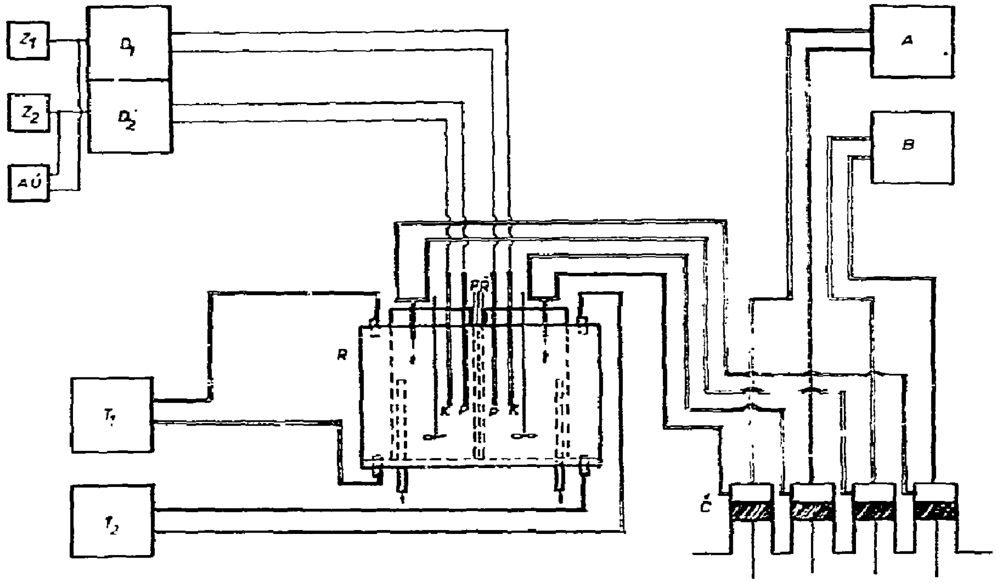


Fig. 1. Experimental set up of two coupled flow-through stirred cells with exchange of mass. R: cells (reactors); PR: three plates dividing the reactors, the middle one perforated; K: calomel electrodes; P: platinum electrodes; A, B: storage of feed solutions; C: pumps; T_1, T_2: independent temperature control in reactors; D_1, D_2: processing of the electrical signal; Z_1, Z_2: line recorders; AU: digital data recording.

through a perforated wall were chosen as a model open system. The Belousov reaction, oxidation of malonic acid by bromate in the presence of ceric/cerous ions and sulfuric acid, studied in detail by Noyes and coworkers [10], was used as model chemical system. Under specified conditions undamped continuous oscillations of characteristic components (ceric/cerous ions, bromide ions) with well defined constant period, sharply dependent on temperature, can be observed in the single cells. When the exchange of mass between cells is started under specified conditions on the degree of interaction and the difference between periods of oscillators (given by difference in cell's temperature), different regimes are set up. Several such regimes will be reported in the subsequent part of the paper.

2. Experimental

In fig. 1 the experimental set up of two connected reactors is shown schematically. Concentration changes of Ce^{3+}/Ce^{4+} ions were followed by recording the redox potential in the system which is proportional to the ratio given above [11]. Pt electrodes together with reference calomel electrodes with salt bridges were used. Both reactors were machined from plexiglass, had an active volume of 150 ml and were stirred in such a way, that small changes in stirring did not affect the course of oscillations (approx. 500 rpm). The same values of concentration of reaction components were always used in both reactors. The reactors were separated by three removable plates and the middle one was perforated with a variable number of holes. Both heating jacket and immersed glass coil were used for the temperature control of the reaction mixture. Temperatures of the reaction mixtures in both reactors could thus be kept constant (nonoscillatory), and at different levels. Four dosing pumps were used for inlet of the reaction mixture separately into both reactors and a constant level of reaction mixture was maintained in the reactors. To start the experiments, the reactors were filled with the reaction mixture and the reaction was let to proceed batchwise approximately for ten minites. Then dosing of reaction mixture with the same

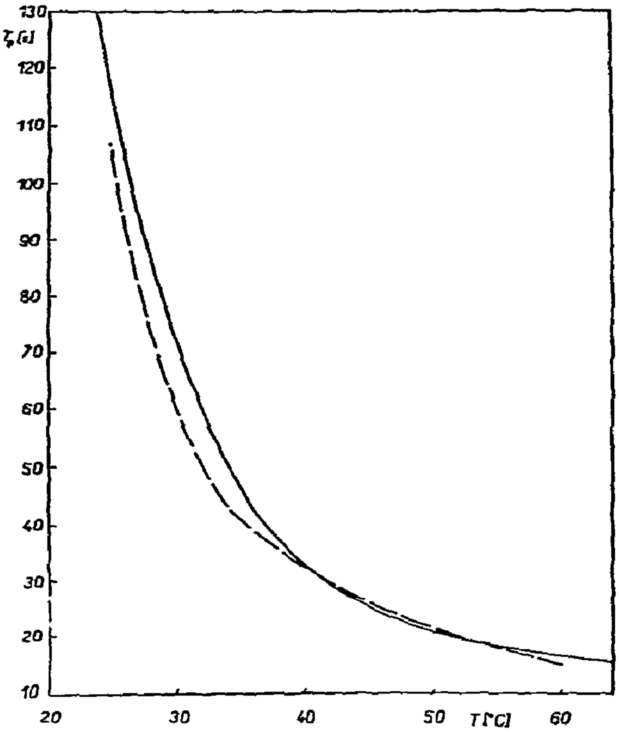


Fig. 2. Dependence of period of oscillations (T_p) on temperature. Concentrations of active components: 0.05 M $KBrO_3$; 0.001 Ce^{4+}; 0.05 M malonic acid; 3N H_2SO_4. (———) Temperature decreased, (– – –) temperature increased.

concentration as the concentration in the reactor was started (two streams separately into each reactor, one containing cerous ions and sulfuric acid, the other malonic acid and bromate). When the regime of oscillations stabilized, two outer plates shielding the dividing perforated plate were removed and the course of oscillations and temperatures in both connected reactors was recorded. Line recorders as well as a digital voltmeter connected with a printer were used for recording. The temperature was used for setting frequency of oscillations at the chosen different values in the two reactors. The typical dependence of the period of oscillations on temperature is shown in fig. 2. The extent of exchange of mass through the perforated plate between the reactors was characterized by the coefficient K (see mass balances given later), determined by using response techniques with NaCl as a tracer [12].

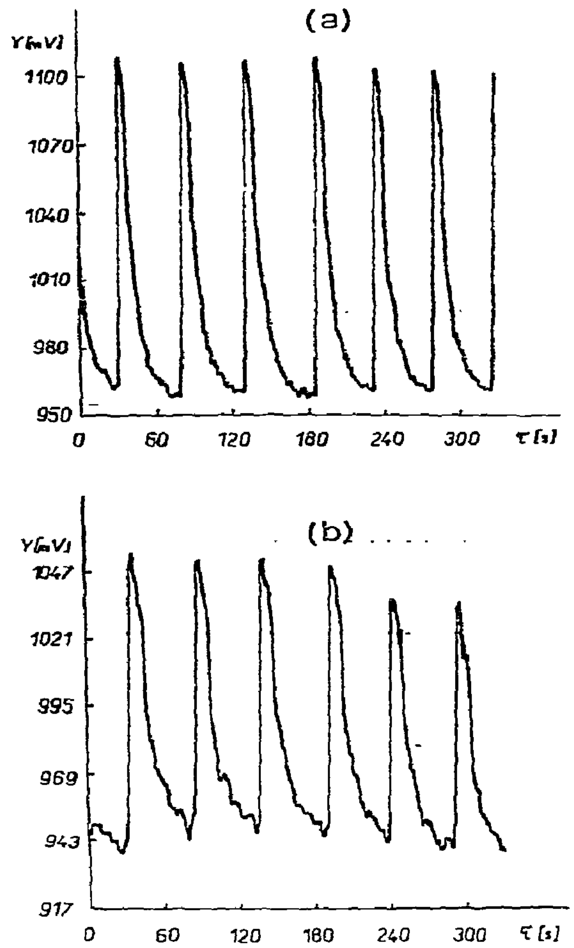


Fig. 3. Synchronization of oscillations in two coupled cells. Dependence of concentration ratio Ce^{4+}/Ce^{3+} on time. Concentrations of active components: 0.01 M $KBrO_3$; 0.001 M Ce^{4+}; 0.032 M malonic acid; 3 N H_2SO_4. $Y \sim Ce^{4+}/Ce^{3+}$; residence time $\bar{\theta} = 29.8$ min; $K = 0.0229$. (a) Temperature $T = 52.5$ °C; original period $T_1 = 52$ s. Synchronized period $T' = 48$ s. (b) Temperature $T = 46$°C; original period $T_2 = 98$ s. Synchronized period $T_2' = 48$ s.

3. Results

3.1. Synchronization at the driving frequency

If the difference between frequency of oscillations at given intensity of interaction (relatively high) in the two reactors is small, synchronization at the same frequency of oscillations (driving frequency) in both reactors can occur. This situation is illustrated in fig. 3 (original recordings are presented in the figures). The concentration values are given in the figure captions; the residence time in both reactors was 29.8 min. The original period of oscillations of the first reactor (without interaction) was $T_1 = 52$ s, of the second reactor $T_2 = 98$ s. When the oscillatory regime in the interacting reactor became stationary, frequencies in both reactors were the same and equal to $T'_1 = T'_2 = 48$ s. Temperatures in both reactors were constant and different during oscillations. Thus both reactors were synchronized at the highest oscillation frequency. The oscillations, however, occurred at different levels of the redox potential (i.e., around different steady states) and with differing amplitude of oscillations, as they correspond to different temperatures in the reactors. If single non-interacting reactors were operated at given stationary temperatures, periods of oscillations were recorded as $T_1 = 80$ s and $T_2 = 120$ s. A number of similar examples for different values of concentrations and temperatures were observed [12].

3.2. Synchronization at a multiple of the driving frequency

Ruelle [5] has predicted that when the original periods of two oscillating systems are T_1 and T_2 then synchronization can occur at the values nT_1 and mT_2 were $n/m \sim T_1/T_2$. In fig. 4 the results are shown of experiments of which the original periods of oscillation were $T_1 = 14$ s and $T_2 = 22$ s. When synchronization occurred, the stationary values of the periods observed were $T'_1 = 12$ s and $T'_2 = 24$ s, i.e., the higher period was twice as high as the lower one. Hence we can conclude that if the originally higher period of one of the interacting oscillatory systems is close to the small entire multiple of the lower period, then synchronization can occur at that period (relatively weak interactions are considered).

3.3. Irregular synchronization at multiples of driving frequency

When at constant values of the coefficient of interaction the difference between the frequencies of interacting oscillators increases, we can observe that originally synchronized oscillators will partly desynchronize.

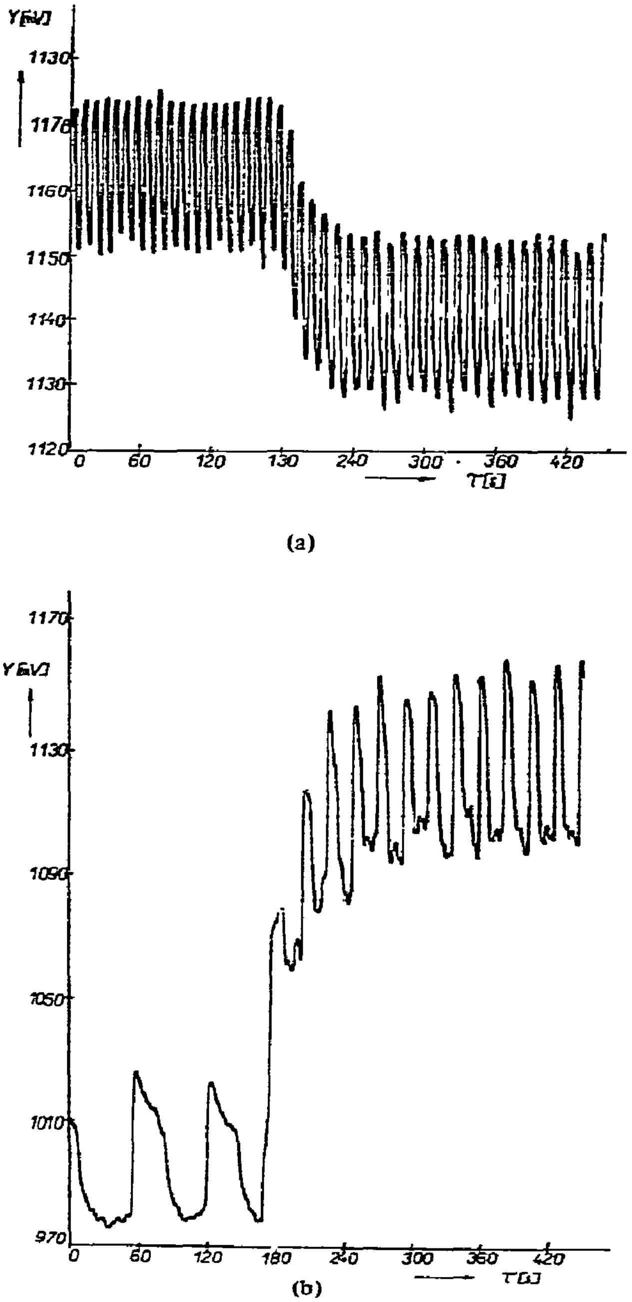


Fig. 4. Synchronization at multiples of driving frequency. Dependence of concentration ratio Ce^{4+}/Ce^{3+} ($\sim Y$) on time. Concentrations of active components: 0.1 M KBrO$_3$; 0.001 M Ce^{4+}; 0.1 M malonic acid; 3 N H$_2$SO$_4$. Residence time $\bar{\theta}$ = 28.8 min. (a) Reactor with higher frequency: original period of oscillations $T_1 = 14$ s; period after synchronization $T'_1 = 12$ s. (b) Reactor with lower frequency: period of oscillation $T'_2 = 2T'_1 = 24$ s.

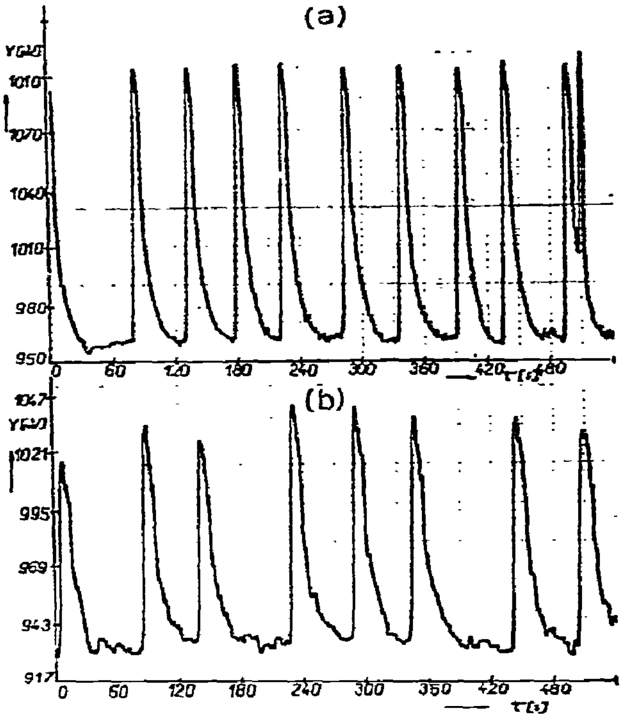


Fig. 5. Irregular synchronization. Dependence of concentration ratio Ce^{4+}/Ce^{3+} (~ Y) on time. Concentrations of active components: 0.01 M KBrO$_3$; 0.001 M Ce^{4+}; 0.032 M malonic acid; 3N H$_2$SO$_4$. Residence time 29.8 min; K = 0.01022. (a) Reactor with higher frequency: temperature T = 55°C. Original period of oscillations T_1 = 70 s; period after synchronization T_1' = 60 s. (b) Reactors with lower frequency: temperature T = 42°C. Original period of oscillations T_2' = 138 s; periods after synchronization T_{21}' = 60 s; T_{22}' = 120 s.

We shall observe two different frequencies in the reactor with lower frequency of oscillations: one which is the same as that for the faster (driving) oscillator and the other which is a small entire multiple of the driving frequency. Such a case is shown in fig. 5. The synchronized frequency is $T_1' = T_{21}' = 60$ s. After desynchronization we can observe two distinct periods of oscillations $T_{21}' = 60$ s and $T_{22}' = 120$ s. When the difference between the original oscillation frequencies is further increased also triple periods $T_{23}' =$ 180 s are observed [12].

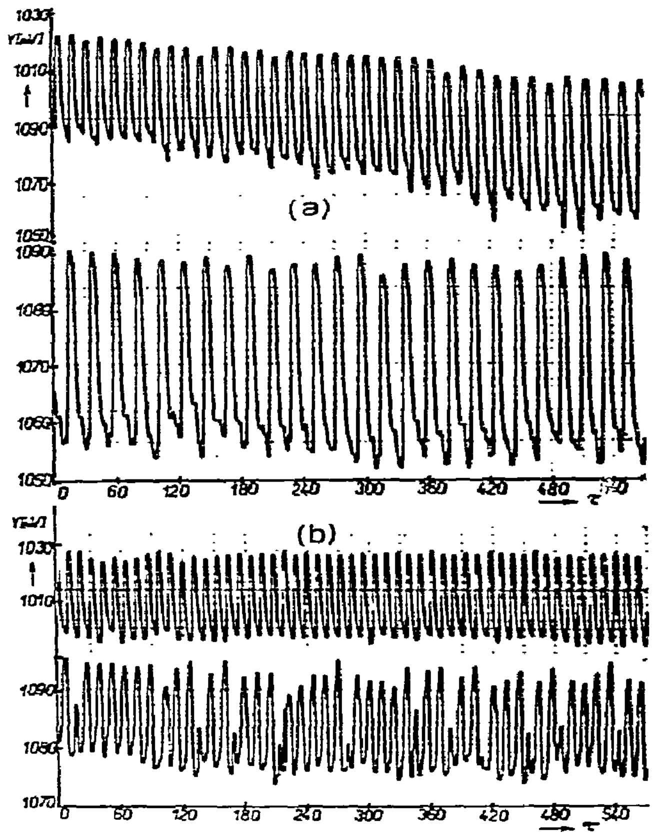


Fig. 6. Rhythm splitting. Dependence of concentration Ce^{4+}/Ce^{3+} (~ Y) on time. Concentrations of active components: 0.1 M KBrO$_3$; 0.001 M Ce^{4+}; 0.1 M malonic aicd; 3N H$_2$SO$_4$; K = 0.0277; residence time $\bar{\theta}$ = 29.8 min. (a) Original course of oscillations (before interaction) T_1 = 14.5 s; T_2 = 19.5 s. (b) Course of oscillations after interaction. Reactor with higher frequency, period after interaction T_1' = 11 s; reactor with lower frequency, period after interaction T_2' = 19.5–11 s.

3.4. Rhythm splitting

When the conditions for synchronization are not fulfilled, we can observe the phenomenon of rhythm splitting: owing to the interaction of two oscillating systems, rhythm (period) in the slower (driven) oscillator is frequently splitted into two parts by action of the faster oscillator. Such a case is shown in fig. 6, where in the upper part of the figure the original (non-interacting) course of oscillations is shown and in the lower part of the figure we can observe irregular splitting of the rhythm of the oscillator with lower frequency. A number of similar cases with various splitting patterns were observed at different values of

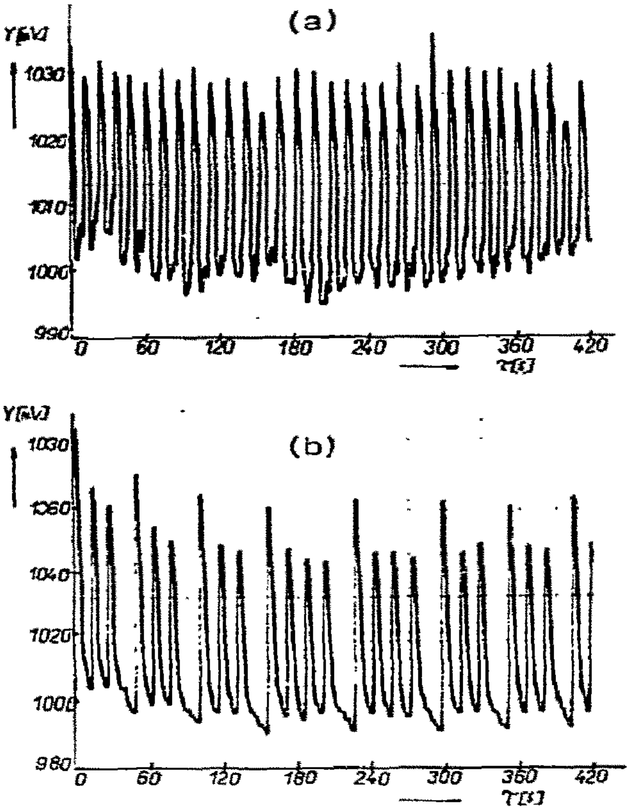


Fig. 7. Amplitude amplification. Dependence of concentration ratio Ce^{4+}/Ce^{3+} (~ Y) on time. Concentrations of active components: 0.1 M $KBrO_3$; 0.001 M Ce^{4+}; 0.1 M malonic acid; 3 N H_2SO_4; residence time $\bar{\theta}$ = 29.8 min; K = 0.0216. (a) Reactor with higher frequency: T = 46.3°C; original frequency of oscillations T_1 = 13 s; period after interaction T_1' = 10 s. (b) Reactor with lower frequency: T = 39°C; original frequency of oscillations T_2 = 23 s; period after interaction T_2' = 10 s – irregular.

concentration and degree of interaction.

3.5. Amplitude amplifications

Under certain conditions, when synchronization of frequencies is nearly achieved and the degree of interaction between the oscillators is relatively high, the phenomenon of amplitude amplification can be observed. In fig. 7 such a case is shown. Here every third (fourth) amplitude of oscillation is amplified for the oscillator which was originally slower. We can see from the figure that the period of oscillation immediately preceding the amplified amplitude is higher than the other periods. This phenomenon can be compared with beats, known, e.g., from studies of heart frequencies.

4. Discussion

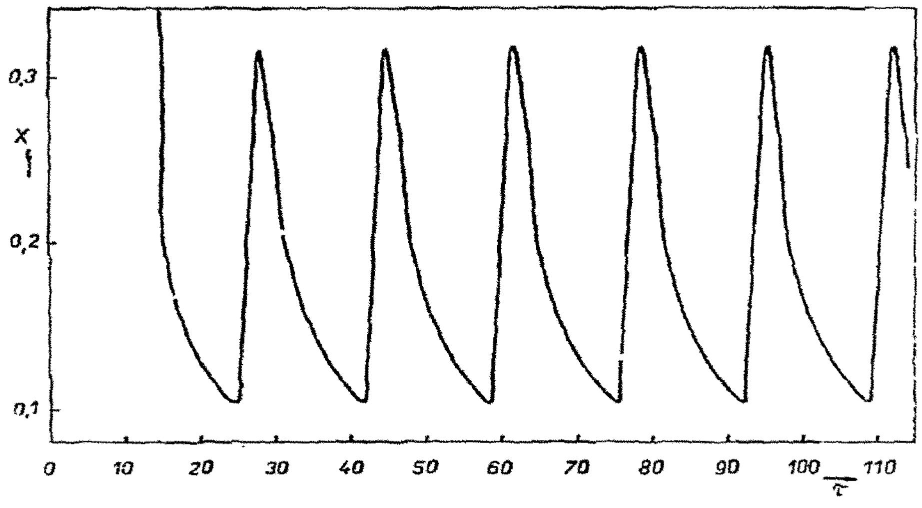
To predict behaviour of coupled chemical reactors one can solve proper mass balance equations with appropriate reaction rate source terms. The kinetic mechanism of the Belousov reaction includes a number of elementary reactions and is not yet fully elucidated [10]. Two mostly semiempirical kinetic models for description of the course of oscillations were used in model studies [12], one proposed by Zhabotinski [13] and the other based on the mechanism described by Noyes and coworkers [10], where three active components describe the course of oscillations. As neither of them was successful in quantitative description of the experimentally observed phenomena, we shall only discuss results obtained with the simpler one.

Zhabotinski et al. [13] proposed the following model for description of the concentration changes of two active reaction components, x (concentration

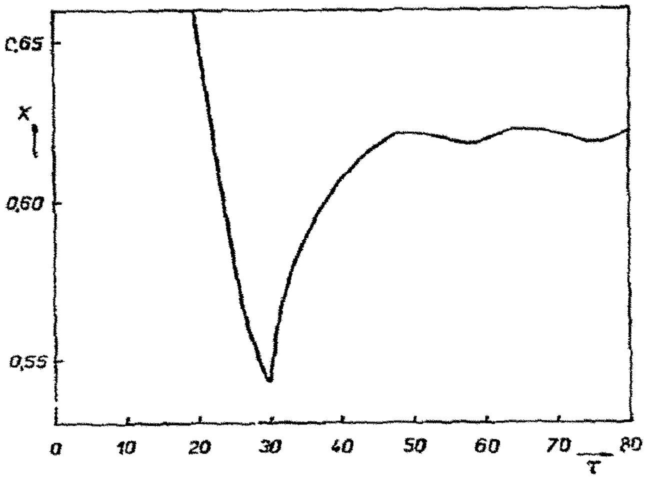
Table 1
Parameters for simulation of interacting reactors

	Temperature	α	β	γ	ϵ	Period calculated for single reactor [5]
Reactor no. 1	46°C	2.5	0.05	0.01212	5×10^{-3}	13.66
Reactor no. 2	32°C	0.6	0.05	0.016	5×10^{-3}	30.3

T_1 experimental ≈ 14 s.
T_2 experimental ≈ 31 s.



(a)



(b)

Fig. 8. Simulated course of oscillations in two interacting cells. Dependence of the dimensionless concentration of Ce^{4+} (x) on time. Concentrations of active components: 0.1 M $KBrO_3$; 0.001 M Ce^{4+}; 0.1 M malonic aicd; 3N H_2SO_4. Experimentally determined periods $T_1' = 14$ s; $T_2' = 31$ s; residence time $\bar{\theta} = 29.8$; $K = 0.01022$. Simulated course of oscillations: (a) $T_1' = 16.8$ s, (b) $T_2' = 17.1$ s.

of ceric ions) and y (concentration of BrO').

$$R_x(x,y) = \beta y(1-x) - \gamma x,$$

$$R_y(x,y) = (\beta/\epsilon)y\{1-x[1+\alpha+(y-\alpha)^2]\}+\beta.$$

These reaction rate expressions can be utilized in constructing the mass balances for the components x and y in isothermal continuous stirred tank reactors with constant volume of reaction mixture. We shall consider two connected reactors (see fig. 1) with inlet of reaction mixture, exchange of reaction mixture between reactors through perforations and outlet of reaction mixture.

Let x_i, $i = 1,2$, denote the dimensionless concentration of Ce^{4+} ions in the first and second reactor respectively and y_i, $i = 1,2$, correspond to the dimensionless concentration of the active intermediates (BrO, in agreement with Zhabotinski). Mass balances can be written in the form

$$\mathrm{d}x_1/\mathrm{d}t = \beta_1 y_1(1-x_1) - \gamma_1 x_1 + K(x_2-x_1) + F(x_{1F}-x_1), \tag{1}$$

$$\mathrm{d}y_1/\mathrm{d}t = (\beta_1/\epsilon_1)y_1\{1-x_1[1+\alpha_1+(y_1-\alpha_1)^2]\} + \beta_1 + K(y_2-y_1) + F(y_{1F}-y_1), \tag{2}$$

$$\mathrm{d}x_2/\mathrm{d}t = \beta_2 y_2(1-x_2) - \gamma_2 x_2 + K(x_1-x_2) + F(x_{2F}-x_2), \tag{3}$$

$$\mathrm{d}y_2/\mathrm{d}t = (\beta_2/\epsilon_2)y_2\{1-x_2[1+\alpha_2+(y_2-\alpha_2)^2]\} + \beta_2 + K(y_1-y_2) + F(y_{2F}-y_2). \tag{4}$$

α, β, γ, ϵ are parameters in the reaction rate expression which have to be fitted to experimental data on oscillations, K describes the extent of interaction between the reactors and F denotes inlet conditions. x_{1F}, y_{1F} and x_{2F}, y_{2F} denote the inlet concentrations of active components. The parameters of chosen experimental example are given in table 1. In fig. 8 the oscillatory course of the concentration of component x for the value of $K = 0.0102\ s^{-1}$ is shown. With increasing value of K, from zero to $K = 0.0102$, the period of oscillations in the first reactor increases and in the second reactor decreases. For $K = 0.0102$ the frequencies of the oscillations are nearly synchronized, however the amplitude of oscillations in the second reactor decreases sharply and the shape of the oscillations is not similar to the experimentally observed oscillations.

The kinetic model of the Belousov reaction has to be improved before more quantitative agreement between experimental and simulated phenomena can be expected.

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